

AMENDMENTS TO THE SPECIFICATION:

Before the paragraph beginning at page 1, line 4,  
insert the following:

--REFERENCE TO COMPUTER PROGRAM LISTINGS

A computer program listing appendix with twelve  
files is provided on a compact disc as part of the invention  
disclosure. The material of the twelve files on the compact  
disc is incorporated by reference. The files included on the  
compact disc are:

- generate-discretization.mu has a size of 6,241 bytes and was  
created (stored on the CD-R) on December 12, 2006.

This file contains the program "generate-discretization",  
referred to as Appendix 1.

- preparations.mu has a size of 3,101 bytes and was created  
(stored on the CD-R) on December 12, 2006. This file contains  
the subroutine "preparations", referred to as Appendix 2.

- setup-equations.mu has a size of 17,414 bytes and was  
created (stored on the CD-R) on December 12, 2006.  
This file contains the subroutine "setup-equations", referred  
to as Appendix 3.

- solve-equations.mu has a size of 4,681 bytes and was created  
(stored on the CD-R) on December 12, 2006. This file contains  
the subroutine "solve-equations", referred to as Appendix 4.

- analyze-solution.mu has a size of 22,654 bytes and was  
created (stored on the CD-R) on December 12, 2006. This file

contains the subroutine "analyze-solution", referred to as Appendix 5.

- appendix6.txt has a size of 10,588 bytes and was created (stored on the CD-R) on December 12, 2006. This file contains the output for D100, Order 2, on the grid  $(-1,1)^3$ , optimize=0, referred to as Appendix 6.
- appendix7.txt has a size of 3,289 bytes and was created (stored on the CD-R) on December 12, 2006. This file contains the output for D100, Order 2, on the grid  $(-1,1)^3$ , optimize=1, referred to as Appendix 7.
- appendix8.txt has a size of 9,465 bytes and was created (stored on the CD-R) on December 12, 2006. This file contains the output for D100, Order 2, on the grid  $(-1,1)^3$ , optimize=2, referred to in the following as Appendix 8.
- appendix9.txt has a size of 47,206 bytes and was created (stored on the CD-R) on December 12, 2006. This file contains the output for D100, Order 4, on the grid  $(-2,2)^3$ , optimize=0, 30, referred to as Appendix 9.
- appendix10.txt has a size of 4,092 bytes and was created (stored on the CD-R) on December 12, 2006. This file contains the output for D100, Order 4, on the grid  $(-2,2)^3$ , optimize=1, referred to as Appendix 10.

- appendix11.txt has a size of 49,617 bytes and was created (stored on the CD-R) on December 12, 2006. This file contains the output for D100, Order 4, on the grid  $(-2,2)^3$ , optimize=2, 5, referred to as Appendix 11.
- appendix12.txt has a size of 16,572 bytes and was created (stored on the CD-R) on December 12, 2006. This file contains the output for D200, Order 2, on the grid  $(-1,1)^3$ , optimize=0, referred to as Appendix 12.--

Please replace the paragraph spanning pages 8 and 9 with the following:

--An example is a general one-dimensional discretization for a derivative at node  $i$  which uses the stencil between the nodes  $i - m$  and  $i + n$ , where  $m$  and  $n$  are given positive integers. On the stencil  $S = (i - m, i - m + 1, \dots, i - 1, i, i + 1, \dots, i + n - 1, i + n)$ , the approximation of the first derivative can be written as

$$u_x = \frac{1}{\Delta x} \{ a_{-m} u_{i-m} + a_{-m+1} u_{i-m+1} + \dots + a_{-1} u_{i-1} + a_0 u_i + a_1 u_{i+1} + \dots + a_{n-1} u_{i+n-1} + a_n u_{i+n} \} \quad (1)$$

The coefficients  $a_j, i - j \in S$   $a_j, j \in S$  determine the numerical properties of the discretization.

A general description of the above discretization can be found in D1 which discusses the method for obtaining the coefficients  $a_j$ . Expression (1) also generalizes to higher derivatives. --

Replace the paragraph beginning at page 47, line 16 with the following:

--The second derivative  $D_2 = \frac{\partial^2 u}{\partial e_1^2}$  can be expressed in the grid-based derivatives  $u_{xx}, u_{yy}, u_{zz}, u_{xy}, u_{yz}$  and  $u_{zx}$  according to

$$\frac{\partial^2 u}{\partial e_1^2} = \cos^2 \alpha \cos^2 \beta \frac{\partial^2 u}{\partial x^2} + \sin^2 \alpha \cos^2 \beta \frac{\partial^2 u}{\partial y^2} + \sin^2 \beta \frac{\partial^2 u}{\partial z^2} + 2 \cos \alpha \cos^2 \beta \sin \alpha \frac{\partial^2 u}{\partial x \partial y} + 2 \cos \alpha \cos \beta \sin \beta \frac{\partial^2 u}{\partial x \partial z} + 2 \sin \alpha \cos \beta \sin \beta \frac{\partial^2 u}{\partial y \partial z} \quad (21)$$

The terms  $T_{xx}, T_{xy}, T_{xz}, T_{yy}, T_{yz}$  and  $T_{zz}$  are obtained by the computer program mentioned before, and are given in appendix 12. The stencils  $T_{xx}, T_{xy}, T_{xz}, T_{yy}, T_{yz}$  and  $T_{zz}$  have been added in various quantities to the stencils  $Tf_{xx}, Tf_{xy}, Tf_{xz}, Tf_{yy}, Tf_{yz}$  and  $Tf_{zz}$  to obtain the more symmetric representation of equation 20. This shows once more the use of the degrees of freedom, and the equivalence between two expressions for an approximation of  $D_p$  using a different  $Tf$  but sharing the basis described by the stencils.--

Replace the list of symbols beginning on page 54 with the following:

--List of symbols

- A an amplitude of a Fourier component
- $A', A''$  intermediate bases in the transformation from the grid basis to the local basis  $B$
- $a, b, c, \dots$  components of the vector  $\vec{a}$
- $\vec{a}$  vector of preferential direction
- $B(\vec{e}_1, \vec{e}_2, \vec{e}_3, \dots)$  a local basis, with  $\vec{e}_1$  along a preferential direction, i.e.  $\vec{e}_1 // \vec{a}$
- $C_c$  computational coefficients used in the approximation of  $D_p^A$ , which are dependent on the numerical formulation used
- $C_s$  computational coefficients used in the approximation of  $D_p^A$ , in the Finite Difference formulation ; weighting coefficients
- $C_{l,m,n,\dots}$  the weighting coefficients  $C_s$  for node  $l, m, n, \dots$
- $D_p$  spatial  $p^{th}$  derivative, to be discretized
- $D_p^A$  an approximation to  $D_p$
- $D_p^{LC}$  an approximation to  $D_p$  in the Finite Difference formulation, representing a linear combination of values

$D_p^{\alpha_i}$	an approximation to $D_p$ in the Distribution Method, depending on the distribution coefficients $\alpha_i$
$D_p^{\varphi,\psi}$	an approximation to $D_p$ in the Finite Element formulation depending on the basis function $\varphi$ and the test function $\psi$
$f$	the flux
$I$	the imaginary unit, such that $I^2 = -1$
$i, j, k, \dots$	indices numbering the nodes of a structured grid
$i_{\max}, j_{\max}, k_{\max}$	maximum indices of a grid
$I_{el}$	the integral of the derivative $D_p$ over a volume, used in the Residual Distribution Method
$M$	order of the error of a discretization
$N$	number of dimensions
$P$	the point where the derivative is computed
$p$	index for a higher ( $p^{th}$ ) derivative, or first derivative ( $p = 1$ )
$p_1, p_2, p_3, \dots$	the powers of the derivatives with respect to $\bar{e}_1, \bar{e}_2, \dots$ in a mixed derivative
$q_1, q_2, q_3, \dots$	arbitrary variables
$r$	an integer summation index
$r_{\max}$	the maximum value of $r$ in the summation
$S$	the stencil : the set of points used in the computation of the approximation $D_p^A$
$t$	the time coordinate
$t_{11}, t_{1,2}, \dots$	coefficients used in the transformation between coordinate systems
$T_\beta$	represent the terms in the discretization $\beta$ resulting from degrees of freedom which remain when the approximated value $D_p^A$ is optimized
$u$	unknown at a grid point
$u_s$	unknown at a point of the stencil $S$
$u_\alpha$	derivative of $u$ with respect to $\alpha$ , e.g. $u_\alpha = \frac{\partial u}{\partial \alpha}$
$\bar{x}$	$N$ -dimensional position vector
$\bar{\nabla}$	differential operator, $\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \dots\right)^T$

$\alpha_i$  the distribution coefficient used in the Distribution Method  
 for the distribution of the part  $\alpha_i I_{el}$  or  $\alpha_i f$  to node  $i$

$\Delta x, \Delta y, \Delta z, \dots$  the mesh spacings in the coordinate directions

$\Delta t$  the time increment

$\frac{\partial u}{\partial x}$  partial  $p^{th}$  derivative ~~with~~ of  $u$  with respect to  $x$

$\frac{\partial^p}{\partial x^p}$  derivative with respect to  $x$

$\varepsilon_n$  error term in  $D_p^A$

$\varepsilon_s$  error term in the expression of  $u_s$  using a truncated Taylor series expansion

$\vec{k}$  the wave number vector

$\varphi$  the basis function used in the Finite Element method for representing  $u$  over the element

$\psi$  test function used in the integrals of the derivative in the Finite Element formulation

$\omega$  the angular frequency—